A Simulation Study for the Evaluation of a Novel Class of Statistical Tests

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Overview

Our tests consider a difference in quantiles as well as stochastic inequality between two samples of data.
The simulation study allows us to compare the performance of our tests to the canonical Brunner-Munzel and Student's T tests through the use of Monte Carlo simulations and Bootstrapping. We have created power curves and box plots of the type I error rate under the null hypothesis to identify situations where our tests outperform the traditional tests.

Bootstrap & Monte Carlo

Monte Carlo: In this context, Monte Carlo simulations are used to provide the test statistics for the tests while being more accurate than asymptotic tests.

Bootstrap: We use bootstrap methods to obtain the test statistics for our data. Specifically, we do random resampling with replacement of a set of data to obtain many distributions from each of which we obtain the appropriate test statistic. We then examine the distribution of these bootstrapped test statistics to make insights into the performance of the test.

Simulation Setup

In this study, we examined many combinations of different parameters which define the circumstances for the tests, specifically we varied: Sample sizes, quantiles under examination, constant multiple of Db and Kd, and the distribution the data is sampled from.

The Tests

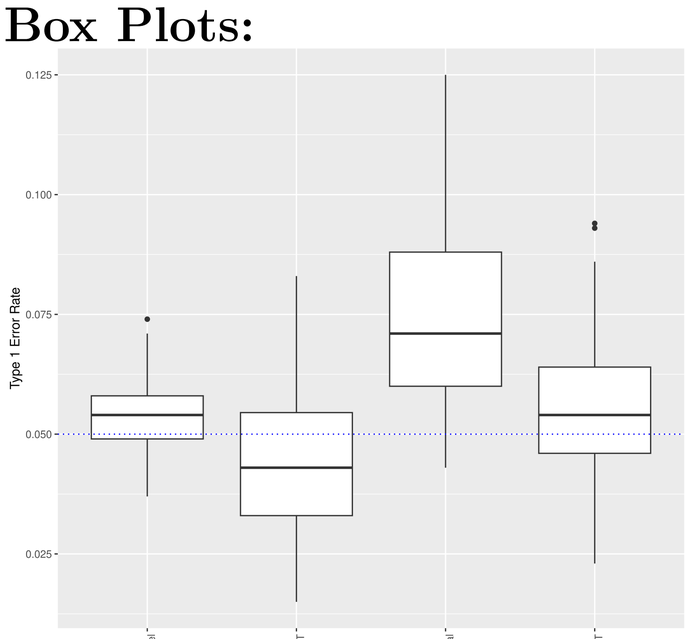
The Tests:

- **Brunner-Munzel:** traditional test for relative effect
- **Student's T:** traditional test for quantile difference
- **Max T:** takes the maximum of the absolute value of the test statistics associated with quantile difference and relative effect.
- **Normal Approximation:** ...

Power Analysis

Here we consider the performance of the test in the case of the two-parameter exponential distribution with a quantile of 0.632.

Box Plots:

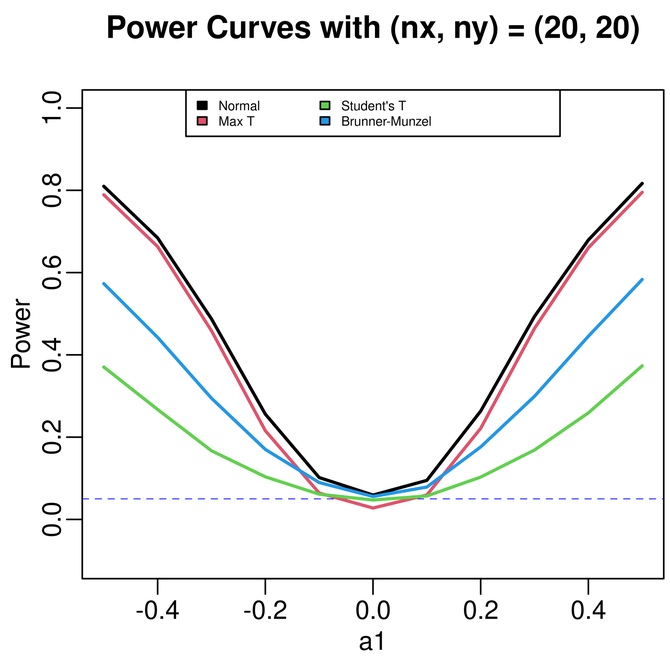


The figure depicts the type I error rate under the null hypothesis for each test under various combinations of parameters.
The following conversion chart is true for both power curve plots:

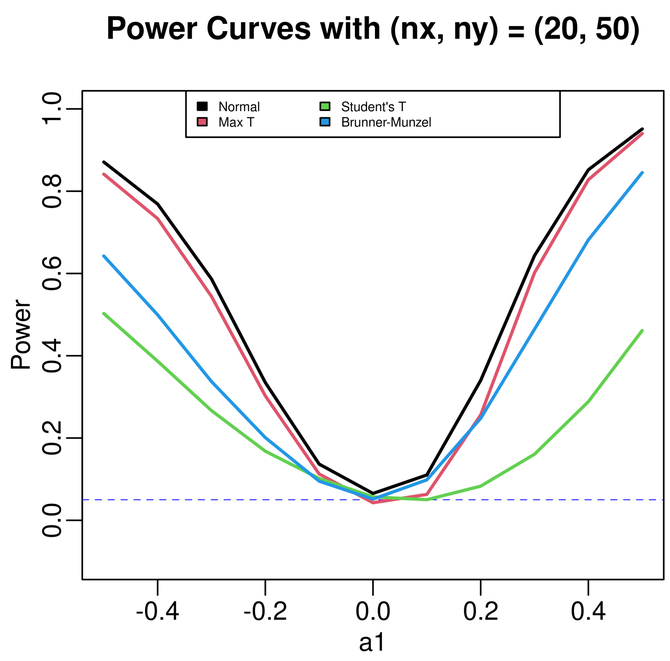
a1	-0.4	-0.2	0.0	0.2	0.4
Ly	0.4	0.2	1	-0.2	-0.4
Sy	1	1	1	1	1

Power Analysis

Even Samples:



Uneven Samples:



Conclusion & Future Work

Max T is slightly conservative in the two parameter exponential distribution but has higher overall power than the traditional tests. More work is needed to understand the behavior of the tests on the Gamma Distribution.